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Technical Notes

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Conduction–Radiation Interaction in Absorbing, Emitting, and Anisotropically Scattering Media with Variable Thermal Conductivity

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Introduction

THE analysis of combined conduction and radiation heat transfer in an absorbing, emitting, and anisotropically scattering medium has many engineering applications and thus has been the subject of numerous investigations as discussed in recent reviews of the literature.[1,2] However, most available studies on this subject consider only constant thermal conductivity and have neglected the effect of the temperature dependence of thermal conductivity which is important for most nonmetal materials when subjected to intermediate or large temperature difference.[3] The primary purpose of this paper is to present the effects of the temperature dependence of thermal conductivity on combined conduction and radiation problems using the differential-discrete-ordinate (DDO) method.[4] The DDO method uses a discrete-ordinate technique to reduce the integro-differential equation of radiative transfer to a system of ordinary differential equations. Unlike the conventional methodology, the resulting set of coupled first-order ordinary differential equations is solved as a two-point boundary value problem. The effects of the temperature-dependent thermal conductivity and several parameters are investigated.

Analysis

For a gray and homogeneous medium confined between two isothermal plates of infinite extent, the dimensionless one-dimensional energy equation for simultaneous conduction and radiation heat transfer is taken as

$$\frac{d^2\theta}{d\tau^2} = \frac{1-\omega}{N(\theta)}\left[\theta^4 - \frac{1}{2}\int_{-1}^{1}\psi(\tau,\mu)\,d\mu\right] - \frac{\gamma}{N(\theta)}\left[\frac{d\theta}{d\tau}\right]^2 \tag{1}$$

with the boundary conditions

$$\theta(0) = 1, \qquad \theta(\tau_0) = \theta_2 \tag{2}$$

Here we have used the following dimensionless quantities:

$$N(\theta) = \frac{\sigma_e k}{4n^2\sigma T_1^3} = \frac{\sigma_e k_0}{4n^2\sigma T_1^3} + \frac{\sigma_e \gamma' T}{4n^2\sigma T_1^3} = N_0 + \gamma\theta \tag{3a}$$

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is the conduction-to-radiation parameter

$$Q^r(\tau) = \frac{q^r(\tau)}{n^2\sigma T_1^4/\pi} \tag{3b}$$

is the dimensionless radiative heat flux

$$\psi(\tau,\mu) = \frac{I(\tau,\mu)}{n^2\sigma T_1^4/\pi} \tag{3c}$$

is the dimensionless intensity

$$\theta(\tau) = T(\tau)/T_1 \tag{3d}$$

is the dimensionless temperature, and $\tau = \sigma_e y$ is the optical depth, and ω the single scattering albedo. In Eq. (3a), the linear relationship, $k = k_0 + \gamma T$, is used to investigate the effects of variable conductivity. The dimensionless intensity ψ satisfies the radiative transfer equation (RTE)

$$\mu\frac{d\psi(\tau,\mu)}{d\tau} = (1-\omega)\theta^4(\tau) - \psi(\tau,\mu) + \frac{\omega}{2}\int_{-1}^{1}\psi(\tau,\mu')p(\mu',\mu)\,d\mu' \tag{4}$$

with the boundary conditions

$$\psi(0,\mu) = \epsilon_1 + \rho_1^s\psi(0,-\mu) + 2\rho_1^d\int_0^1\psi(0,\mu')\mu'\,d\mu', \quad \mu > 0 \tag{5a}$$

$$\psi(\tau_0,-\mu) = \epsilon_2\theta_2^4 + \rho_2^s\psi(\tau_0,\mu) + 2\rho_2^d\int_0^1\psi(\tau_0,\mu')\mu'\,d\mu', \quad \mu > 0 \tag{5b}$$

where ϵ is the emissivity, and ρ^s and ρ^d are the specular and diffuse reflectivities, respectively.

Equations (1) and (4) are coupled through θ and ψ and have to be solved simultaneously. It is very difficult to obtain analytic solutions of this very complicated nonlinear system. In this work, the numerical DDO method is utilized to solve the RTE. After replacing the integrals over μ' in Eqs. (4), (5a), (5b) by quadratures, Eq. (4) is reduced to the following system of ordinary differential equations[4]

$$\mu_i\frac{d\psi_i(\tau)}{d\tau} = (1-\omega)\theta^4(\tau) - \psi_i(\tau) + \frac{\omega}{2}\sum_{j=-M}^{M} w_j\psi_j(\tau)\Phi_{ji}, \qquad i = -M,\ldots,M \tag{6}$$

where μ_i values are the quadrature points, w_i values the corresponding weights, $\psi_i(\tau) = \psi(\tau,\mu_i)$, and $\Phi_{ji} = \Phi(\mu_j \rightarrow \theta_i)$. In this work, both linearly anisotropic scattering and Rayleigh scattering are considered to examine the effects of scattering anisotropy. The Rayleigh scattering phase function is ex-

Table 1 Comparison of Q^c, Q^r, and Q^t distributions for isotropic scattering and constant N at $\epsilon_1 = \epsilon_2 = \tau_0 = 1$, $\theta_2 = 0$, and several values of ω

			Q^c			$Q^r/4\pi N$		
N	ω	Q^t	$\tau = 0.0$	$\tau = 0.5$	$\tau = 1.0$	$\tau = 0.0$	$\tau = 0.5$	$\tau = 1.0$
0.5	0.0[a]	1.2981	0.9396	0.9879	1.1447	0.3585	0.3102	0.1534
	[b]	1.2981	0.9395	0.9880	1.1446	0.3586	0.3101	0.1535
	0.5[a]	1.2884	0.9491	0.9930	1.0983	0.3392	0.2954	0.1900
	[b]	1.2888	0.9463	0.9930	1.1010	0.3425	0.2958	0.1878
	0.9[a]	1.2793	0.9798	0.9985	1.0305	0.2995	0.2803	0.2488
	[b]	1.2795	0.9798	0.9985	1.0305	0.2997	0.2810	0.2490
	1.0[a]	1.2767	1.0000	1.0000	1.0000	0.2767	0.2767	0.2767
	[b]	1.2769	1.0000	1.0000	1.0000	0.2769	0.2769	0.2769
0.05	0[a]	4.0530	0.8986	0.7159	2.2187	3.1544	3.3371	1.8344
	[b]	4.0518	0.8921	0.7166	2.2190	3.1597	3.3352	1.8328
	0.5[a]	3.8845	0.7908	0.8314	1.8689	3.0937	3.0531	2.0156
	[b]	3.9351	0.7822	0.8326	1.8702	3.1529	3.1026	2.0649
	0.9[a]	3.7991	0.8382	0.9745	1.2877	2.9608	2.8246	2.5114
	[b]	3.8007	0.8374	0.9746	1.2876	2.9633	2.8262	2.5131
	1.0[a]	3.7670	1.0000	1.0000	1.0000	2.7670	2.7690	2.7690
	[b]	3.7690	1.0000	1.0000	1.0000	2.7690	2.7690	2.7690

[a]Lii and Ozisik.[5]
[b]Present study.

Table 2 Effects of variable thermal conductivity on radiation, conduction, and total heat flux distributions at $\tau_0 = 1$, $\theta_2 = 0.5$, $\epsilon_1 = \epsilon_2 = 1$ for several values of ω

			ω			ω		
N_0	γ	τ/τ_0	0	0.5	1.0	0	0.5	1.0
				$Q^r/4\pi$			$N \times Q^c$	
0.5	0.4	0.0	0.1374	0.1378	0.1297	0.4061	0.4000	0.4000
		0.25	0.1645	0.1520	0.1297	0.3791	0.3858	0.4000
		0.5	0.1559	0.1453	0.1297	0.3876	0.3925	0.4000
		0.75	0.1298	0.1277	0.1297	0.4138	0.4101	0.4000
		1.0	0.0955	0.1062	0.1297	0.4480	0.4316	0.4000
		$Q^t \times N$	0.5436	0.5378	0.5297			
0.8	0	0.0	0.1410	0.1405	0.1297	0.4021	0.3971	0.4000
		0.25	0.1666	0.1534	0.1297	0.3765	0.3841	0.4000
		0.5	0.1547	0.1446	0.1297	0.3884	0.3929	0.4000
		0.75	0.1267	0.1256	0.1297	0.4164	0.4119	0.4000
		1.0	0.0928	0.1041	0.1297	0.4504	0.4335	0.4000
		$Q^t \times N$	0.5431	0.5375	0.5297			
0.5	−0.4	0.0	0.1503	0.1481	0.1297	0.0905	0.0879	0.1000
		0.25	0.1668	0.1550	0.1297	0.0739	0.0810	0.1000
		0.5	0.1491	0.1408	0.1297	0.0917	0.0952	0.1000
		0.75	0.1204	0.1203	0.1297	0.1203	0.1157	0.1000
		1.0	0.0880	0.0993	0.1297	0.1527	0.1367	0.1000
		$Q^t \times N$	0.2407	0.2360	0.2297			
0.2	0	0.0	0.1371	0.1379	0.1297	0.1059	0.0996	0.1000
		0.25	0.1618	0.1512	0.1297	0.0811	0.0864	0.1000
		0.5	0.1543	0.1447	0.1297	0.0885	0.0928	0.1000
		0.75	0.1309	0.1279	0.1297	0.1120	0.1097	0.1000
		1.0	0.0971	0.1065	0.1297	0.1458	0.1310	0.1000
		$Q^t \times N$	0.2429	0.2375	0.2297			

pressed as

$$\Phi_{ji} = 1 + \frac{1}{8}(3\mu_j^2 - 1)(3\mu_i^2 - 1) \tag{7}$$

The linearly anisotropic scattering phase function is expressed as $\Phi_{ji} = 1 + A_1\mu_j\mu_i$, where A_1 is constant. $A_1 \to 1$ represents strong forward scattering, whereas $A_1 \to -1$ corresponds to strong backward scattering. Similarly, Eq. (1) is reduced to

$$\frac{d^2\theta}{d\tau^2} = \frac{1-\omega}{N(\theta)}\left[\theta^4 - \frac{1}{2}\sum_{j=-M}^{M} w_j\psi_j(\tau)\right] - \frac{\gamma}{N(\theta)}\left[\frac{d\theta}{d\tau}\right]^2 \tag{8}$$

Equations (6–8) constitute a system of 2 M first-order ordinary differential equations in ψ_i and a second-order nonlinear ordinary differential equation in θ. The 2 M boundary conditions for ψ_i are supplied by

$$\psi_i(0) = \epsilon_1 + \rho_1^s\psi_{-i}(0) + 2\rho_1^d \sum_{j=1}^{M} w_j\psi_{-j}(0)\mu_j$$

$$i = 1, \ldots, M \tag{9a}$$

$$\psi_{-i}(\tau_0) = \epsilon_2\theta_2^4 + \rho_2^s\psi_i(\tau_0) + 2\rho_2^d \sum_{j=1}^{M} w_j\psi_j(\tau_0)\mu_j$$

$$i = 1, \ldots, M \tag{9b}$$

and the boundary conditions for θ are specified by Eq. (2).

The coupled nonlinear ordinary differential equations are solved by using subroutine BVPFD of the IMSL software package. The subroutine BVPFD solves two-point boundary value problems using a variable order, variable step-size finite difference method with deferred corrections. The convergence criterion in all of the calculations was chosen such that a relative tolerance of no greater than 10^{-3} was met at all mesh points. The results were obtained by carrying out the numerical algorithm on a VAX 8800 at National Chiao Tung University. Once the temperature and radiative intensity distributions are known, the radiative heat flux is obtained from the relation

$$Q^r(\tau) = 2\pi \sum_{j=-M}^{M} w_j \psi_j(\tau) u_j \qquad (10)$$

and the total heat flux is given by

$$Q^t(\tau) = \frac{q^t(\tau)}{k\sigma_e T_1} = -\frac{d\theta(\tau)}{d\tau} + \frac{Q^r(\tau)}{4\pi N(\theta)} \qquad (11)$$

In present work, results are obtained with 16 quadrature points and 41 uniformly spaced grids.

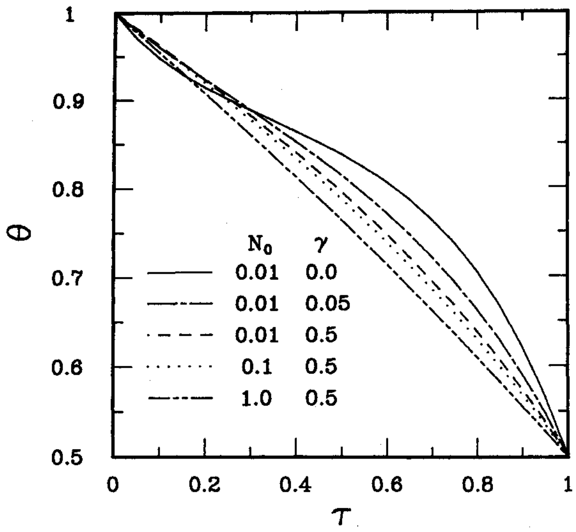

Fig. 1 Effects of variable thermal conductivity on dimensionless temperature distribution at $\epsilon_1 = \epsilon_2 = 1$, $\tau_0 = 1$, $\theta_2 = 0.5$, and $\omega = 0.8$.

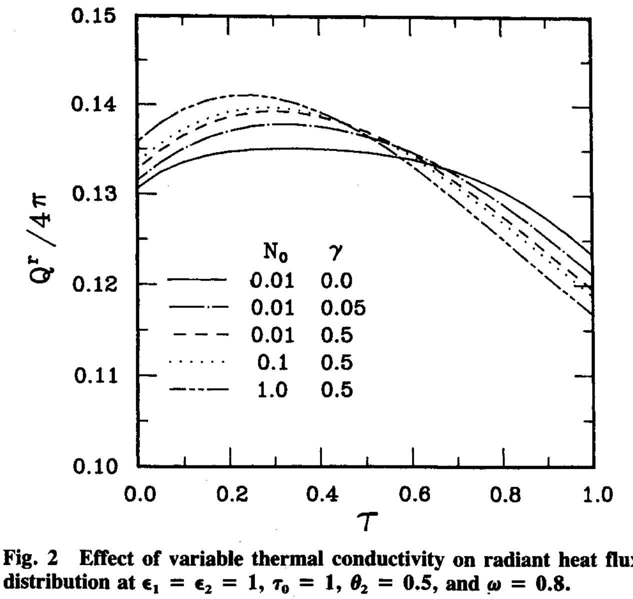

Fig. 2 Effect of variable thermal conductivity on radiant heat flux distribution at $\epsilon_1 = \epsilon_2 = 1$, $\tau_0 = 1$, $\theta_2 = 0.5$, and $\omega = 0.8$.

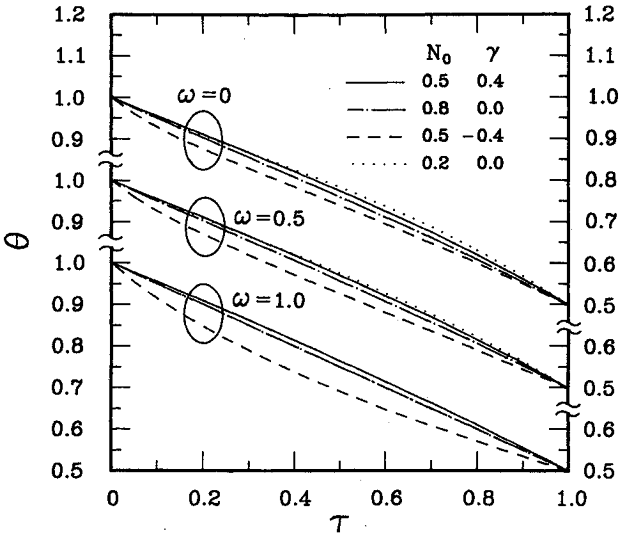

Fig. 3 Effect of variable thermal conductivity on dimensionless temperature distribution at $\epsilon_1 = \epsilon_2 = 1$, $\tau_0 = 1$, $\theta_2 = 0.5$ for several values of ω.

Results and Discussion

In order to verify the validity of present numerical solutions, the heat flux results of the present study are compared with those of Lii and Özisik.[5] Table 1 presents the comparisons of dimensionless heat fluxes, Q^t, Q^r, and Q^c, under various conditions. It is readily observed that the results are in excellent agreement for all cases. Figures 1 and 2 demonstrate the effects of variable thermal conductivity on dimensionless temperature and radiative heat flux distributions at $\epsilon_1 = \epsilon_2 = 1$, $\tau_0 = 1$, $\theta_2 = 0.5$, and $\omega = 0.8$. Note in Fig. 2, $Q^r/4\pi$ is used instead of $Q^r/4\pi N$ because $N(\theta)$ is not constant under present consideration. It can be observed that, as N_0 or γ is increased, the temperature distribution is more linear, whereas the radiant heat flux distribution is more nonuniform. This is mainly due to the effects of average N, N_{av} of the slab as depicted in Fig. 3 and Table 2. While Table 2 displays the effects of variable conductivity on radiation, conduction, and total heat flux distributions at $\tau_0 = 1$, $\theta_2 = 0.5$, $\epsilon_1 = \epsilon_2 = 1$ for various values of ω, Fig. 3 demonstrates the effects on dimensionless temperature distributions for the cases. The total heat flux for $N = 0.5 + 0.4\theta$, as shown in Table 2, is nearly equal to that for $N = 0.8 =$ const, and is exactly equal to when $\omega = 1$. The same is true for $N = 0.5 - 0.4\theta$ and $N = 0.2 =$ const. The reason is that N_{av} is approximately 0.8 for $N = 0.5 + 0.4\theta$ and 0.2 for $N = 0.5 - 0.4\theta$ due to the nearly linear temperature distribution. It can also be observed that, for same N_{av}, the radiant heat flux distribution of the variable conductivity case is more uniform than the constant conductivity case, whereas the dimensionless temperature distribution of the constant conductivity case is more linear than the variable conductivity one. Examining Fig. 3 also reveals that a positive γ causes the temperature of the medium to increase, and a negative γ tends to decrease the temperature. This phenomenon is more evident as ω is increased.

Concluding Remarks

The problem of simultaneous conduction and radiation heat transfer through absorbing, emitting, and anisotropically scattering media with temperature-dependent thermal conductivity has been investigated. Results of numerical analysis in present study can be summarized as:

1) The temperature dependence of thermal conductivity has significant effects on heat flux and temperature distribution. Positive γ will increase the temperature, while negative γ will decrease the temperature.

2) The effects of variable conductivity is more significant as ω is increased.

3) The DDO method is an accurate and powerful method to deal with the radiative transfer problems.

Acknowledgment

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Second-Law Optimization of Forced Convection of Non-Newtonian Fluids in Ducts

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Nomenclature

A = parameter, mkT_0/q''^2, $(s)^{n+1}$
B = U/b for parallel plates, $(s)^{-1}$ or U/R for circular duct, $(s)^{-1}$
b = one half the plate spacing, m
c_p = specific heat at constant pressure, kJ/kg K
k = thermal conductivity, W/m K
$\dot{M}$ = mass flow rate, kg/s
m = average consistency index, Ns^n/m^2
N_s''', N_s' = entropy generation number
n = power law index
Pe = Peclet number, Ub/α for parallel plates or UR/α for circular duct
q'' = heat flux, W/m^2
R = radius of the circular duct
r = radial coordinate, m
r_* = r/R, nondimensional radial coordinate
$\dot{S}'''_{gen}$, $\dot{S}'_{gen}$ = rate of entropy generation, W/m^3 K and W/m K, respectively
T = absolute temperature, K
T_0 = reference temperature, K
U = average velocity in the direction of flow, m/s
u = x component of the velocity, m/s
$\dot{u}'''$ = volumetric rate of heat generation, W/m^3 K
v = y component of the velocity, m/s
$\boldsymbol{v}$ = velocity vector, m/s
W = width of the channel, m
w = z component of the velocity, m/s
x = x component of the Cartesian coordinate system, m
y = y component of the Cartesian coordinate system, m
y_* = y/b, nondimensional y coordinate
z = z component of the Cartesian coordinate system, m
α = thermal diffusivity, m^2/s
ρ = density, kg/m^3
τ = stress tensor

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I. Introduction

THE utilization of the second law of thermodynamics to rate and optimize engineering applications in which convective heat and mass transfer play a pivotal role has been advocated by energy conservationists and researchers in the past few decades. Most notably, among researchers in the United States, Bejan[1-3] has shown that the second law of thermodynamics "bridges the gap" between convective heat transfer, fluid mechanics, and thermodynamics. Sun et al.[4] were the first to publish expressions for the entropy generation in forced convection between two parallel plates including mass transfer. Sun et al.[5] later generalized their findings by devising a criterion for the entropy generation accounting for the simultaneous existence of heat and mass transfer irreversibilities. Poulikakos and Johnson[6] derived a general expression for the entropy generation for combined heat and mass transfer in external forced convection. Moreover, they applied this expression to optimize the process of heat and mass transfer in forced convection from a flat plate and from a cylinder in crossflow.

With energy conservation as the main incentive, a considerable amount of research has recently been focused on the performance of non-Newtonian fluids in forced convection.[7] Unlike in the case of Newtonian fluids where the entropy generation minimization principle has been used to optimize the thermal design of basic forced convection processes, the second-law–based optimization of forced convection applications of non-Newtonian fluids has escaped the scrunity of the research community. It is the goal of this paper to aim at this deficiency. To this end, first a general expression of entropy generation in convection of power-law fluids is presented. Next, the problems of forced convection of a power-law fluid in a circular pipe and between two parallel plates are analyzed from the viewpoint of the entropy generation minimization principle.

II. Entropy Generation in Forced Convection of Non-Newtonian Fluids

This section contains the foundations of entropy production in a moving non-Newtonian fluid as they exist in the heat transfer and thermodynamics literature. The volumetric entropy generation at any point in a fluid engaged in convective heat transfer is given by

$$\dot{S}'''_{gen} = \frac{k}{T^2}(\nabla T)^2 - \frac{\tau:\nabla \boldsymbol{v}}{T} + \frac{\dot{u}'''}{T} \quad (1)$$

The first term on the right-hand side of Eq. (1) accounts for entropy generation because of heat diffusion (caused by temperature gradients). The second term represents entropy generation due to dissipation caused by viscous and plastic deformation; mechanical energy is degraded irreversibly into thermal energy. The last term on the right-hand side of Eq. (1) accounts for entropy generation due to the presence of